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RIGIDITY IN CHILDREN'S PROBLEM SOLVING.  
BY- CUNNINGHAM, JOHN D.

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RESEARCH FINDINGS RELATED TO RIGIDITY IN CHILDREN'S PROBLEM-SOLVING ACTIVITIES ARE REVIEWED. EINSTELLUNG RIGIDITY WHICH AFFECTS CHILDREN'S PERFORMANCE IN PROBLEM SOLVING IS CAUSED BY THE MATERIALS AND THE SEQUENCE OF EVENTS INVOLVED IN PROBLEM SITUATIONS. FUNCTIONAL FIXATION OCCURS WHEN CHILDREN ARE UNABLE TO USE OBJECTS FOR PROCESSES IN THE SOLUTION OF NEW PROBLEMS DUE TO THEIR PREVIOUS USE IN UNRELATED PROBLEMS. SUSCEPTIBILITY TO RIGIDITY AND ABILITY TO OVERCOME IT HAVE BEEN RELATED TO PERSONALITY TYPES, SITUATIONAL FACTORS, EFFECT OF PREVIOUS EXPERIENCE ON PERCEPTIONS, PSYCHOLOGICAL STRESS, AGE, SEX, AND INTELLIGENCE. FLEXIBILITY IN PROBLEM SOLVING IS A TYPE OF LEARNED BEHAVIOR. STUDENTS EXPOSED TO A VARIETY OF PROBLEMS ARE ABLE TO MAKE A SMOOTHER TRANSITION TO NEW PROBLEMS THAN THOSE THAT HAVE BEEN DRILLED ON MANY SIMILAR PROBLEMS. PERMISSIVE TEACHING SITUATIONS AND SOCRATIC QUESTIONING MAY ALSO CONTRIBUTE TO THE DEVELOPMENT OF FLEXIBILITY IN STUDENTS PROBLEM SOLVING BEHAVIORS. THIS ARTICLE IS PUBLISHED IN "SCHOOL SCIENCE AND MATHEMATICS," VOLUME 66, NUMBER 4, WHOLE 582, APRIL 1966. (AG)

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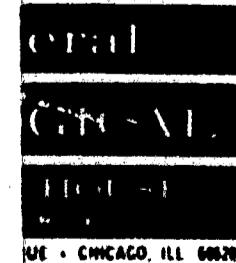
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## Rigidity in Children's Problem Solving\*

John D. Cunningham†

*Science Curriculum Improvement Study and Florida State University,  
Tallahassee, Florida*

The growth within the past decade of so-called "discovery" approaches to teaching, particularly in the sciences, has been explosive. These approaches emphasize "inquiry," require a high degree of flexibility in thinking, and stress, more than did traditional curricula, divergent rather than convergent thought processes. This study originated in an attempt to identify some of the variables affecting rigidity in problem-solving behavior in hope that the exclusion or control of such variables might increase flexibility of thought. Of particular concern were the blinding effects of habit, specifically with what happens when, to quote Luchins, a habit "ceases to be a tool discriminately applied but becomes a procrustean bed to which the situation must conform; when, in a word, instead of the individual mastering the habit, the habit masters the individual" (32).

The problem of behavioral rigidity has been of perennial interest to psychologists. Whether the term *Einstellung-effect*, mental set, disposition, readiness, determining tendency, or other similar term is used, the concept of a predisposition to respond in a given way which can affect perception, reasoning, and memory becomes a very important one. Chown says, "Few major topics in contemporary psychology appear to offer more promise than rigidity, and the amount of work reported on this subject has been increasing year by year" (11).

With their concepts of "fixation" and "style of life," Freud and Adler, respectively, described behavior consistently inappropriate or, at least, responsive only to a limited set of cues in a variety of situations (4, 23). These clinical approaches, however, failed to describe clearly the conditions eliciting the behavior except, perhaps, at a high level of generality.

While the greatest impetus to research in rigidity in American psychology came with the publication of Luchins' original work, numerous earlier investigators hinted at its existence (32). Chant, for example, found that many errors in problem solving were caused by an "interpretive" approach, in which old meanings and interpretations determined the response, rather than a fresh approach to the materials at hand (10).

One of the specific purposes of this study was to extend the under-

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standing of the *Einstellung* phenomenon in children. The term *Einstellung* has been defined as a "relatively rigid and simple attitude or predisposition" (19). The German root means *to put in place* or *to adjust* (beforehand). *Einstellung* is sometimes spoken of as an "objective" set because it is developed by the materials and sequence of events in the experimental situation as distinguished from a "subjective" set, such as one brought to the experiment by the subject.

Guetzkow has separated problem-solving set into the two factors of "susceptibility to set" and "ability to overcome set," and the separation has been largely substantiated by others (3, 16, 24, 35). There have been few significant correlates reported to the first of Guetzkow's factors, i.e., young and old, intelligent and feeble-minded, and males and females all seem about equally susceptible to the establishment of set. The crucial factor seems to be the ability to overcome an established set when conditions allow or make a change more efficient. Several significant correlates of ability to overcome set have been identified in studies of adolescents and adults. In general, males, subjects high in intelligence, and older subjects have been found superior in ability to overcome set than have their counterparts.

While rigidity of thought is evident in many problem situations, the principles underlying and the correlates of rigidity have been discovered largely by the use of a few semi-standardized instruments. One of the best known of these psychometric tools is the Water Jar Test. Basically, subjects are told to imagine themselves near a source of water and, with containers of known maximum volume but with no graduated markings, to obtain a specified volume. Sometimes the test is administered utilizing actual water containers and an actual source of water. An Alphabet Maze that parallels the Water Jar Test is also being perfected. Following practice problems which serve to acquaint subjects with the stimulus elements and the rules for their manipulation, these measures generally consist of two or three types of problems. First, there is a series of set-inducing problems, only solvable by a single complicated method. Next is a series of so-called critical problems solvable by both the set method and by a short, direct method. If instructions call for the subject to indicate only the shortest of the alternatives if ever they discover a problem solvable by more than one method, then the continued use of the set solution on the critical problems may be taken as evidence of the establishment of a set or of the *Einstellung*-effect. Finally, an extinction or test problem, solvable only by a direct method, may be added. Numerous other variations of the problems serve to maximize or minimize the *Einstellung*-effect.

#### RELATIONSHIPS TO RIGIDITY

A number of investigators have sought or suggested relationships existing between a variety of factors and rigidity. Some of the more important such factors to science education are discussed below.

*Rigidity as a Function of Learning.* Some investigators claim that the behavioral changes commonly observed in problem solving have much in common with those observed in simple associative learning. Eysenck, for example, states that:

The development of *Einstellung* rigidity in Luchins' test is clearly a learned phenomenon, very similar to ordinary discrimination learning (i.e., some types of solution, and perhaps perceptions, are rewarded and some are not during the development of the set) (21).

"Functional fixation" is one factor particularly important to science education. Duncker originally proposed that because of the previous use of an object or process with a function dissimilar to that demanded by a present problem, subjects are inhibited in discovering the appropriate new use (18). Others have largely confirmed Duncker's hypothesis and have found that functional fixation decreases with increasing time following initial use of the object (3, 8). The longer the interval and the larger the amount of intervening activity, the greater the amount of retroactive inhibition.

Several investigators have found difficult tasks and massed practice to increase rigid responses as compared to simple tasks and distributed practice (9, 42). Generally the response sets of the highly rigid subjects show greater resistance to extinction than do those of non-rigid subjects (43).

*Personality vs. Situational Factors in Rigidity.* The relationship between personality and rigidity has been the concern of a large number of investigators. The issue is, essentially, whether rigidity is to be regarded as a dimension of a given person's behavior that underlies many diverse actions, from social relationships to mathematical calculations, or as relatively specific to a given situation. Individuals who suffer from a postulated general rigidity syndrome would be expected to show little variability in behavior, to be ethnocentric, and to have few methods available for solving problems. While the results of many studies have been interpreted in favor of such a hypothesis, even more studies cast doubt on such a unitary trait of intellectual rigidity. Benedetti, for example, doubts that there is a generalized rigidity potent enough to manifest itself in many situations that are appreciably different but postulates that there still may be a lower-order vulnerability to sets in situations featuring one or more stimulus-variables which are known to encourage mechanization (7).

Rigidity may be due to any number of factors other than rigidity in personality structure. Some claim that rigidity may be related to a tendency toward conformity, others to intolerance of ambiguity (27, 30, 39). To search for solutions is to be in the midst of ambiguity; to make use of a solution stipulated by authority, such as when using the set solution in Einstellung problems, is to be secure. Levitt and Zelen point out, however, that conformity is not synonymous with rigidity (30). Conformity becomes rigidity only when the standards to which one conforms no longer make for adaptation. Viewing rigidity as possibly the result of a defense against anxiety in which the number of alternatives is reduced because they may pose personal threats, Eriksen and Eisenstein found that subjects who manifested difficulty in overcoming sets tended to avoid speculation as to the nature of an ambiguous stimulus (20).

Early evidence suggesting that situational factors may have a bearing upon behavioral rigidity was reported by Luchins:

In many of the public school classes in which we conducted the experiments much of the child's behavior was circumscribed by fixed rules and procedure. Restraint, pedantry, obedience, submissiveness were desired traits. As a result of these factors, we believe, conditions and attitudes may have been created which furthered the Einstellung responses (32).

Miller likewise found that schools stressing "drill" favored the development of rigid individuals (38).

Situational factors have been widely manipulated in studies. Varied have been the populations tested, the problems, the time factor, instructions, previous and subsequent experiences of subjects, the experimenters, and environmental factors. From these studies it appears that while situational factors exercise a profound influence on the production of rigidity, the effectiveness of any given situational demand as a determiner of rigidity is also related to the general mode of adaptation characteristic of the individual as well as the state of the subject at the time.

*Perceptual Rigidity.* Recent psychological literature witnesses an ever increasing emphasis upon the role of the perceiver in the process of perception and a recognition of the importance of variables other than those inherent in the structure of the stimulus. Many experiments have demonstrated that specified attitudes or preconceptions on the part of the subject result in selective perception.

For example, Cunningham and Karplus studied the effect of children's assumptions upon their perception of falling objects (15). Subjects examined two balls (lead and cork) of equal volume in order to determine their relative and substantially different weight, were told that the balls were to be raised to the ceiling at the same level and dropped simultaneously, and were asked to predict the

relative arrival time at a pan of sand located on a table under the balls. The expectation of most that the heavy ball would hit first so affected the children's observation of the event that they actually claimed to have seen the heavy ball strike the sand first. The children's observations were accepted but the question was put in a new form: Given the heavy object *A* to be released at a certain  $h_A$  above the floor, at what  $h_B$  must the light object *B* be released simultaneously if it is to strike the floor at the same time as *A*? (Since gravity operates irrespective of the weight of objects of like volume, the "correct" answer is, of course,  $h_B = h_A$ .) In some cases, children proposed  $h_B \approx 2/3h_A$ , and when such an experiment was carried out, the arrival of *B* before *A* was easily recognized. This observation led to an adjustment  $h_B \approx 5/6h_A$  and, in many cases,  $h_B = h_A$  after two or more trials. Some children, however, remained blinded by their assumption and were incapable of observing the simultaneous impact of the balls.

Studies employing reversible figures, ambiguous stimuli, or tachistoscopic exposures have shown that a subject's preparatory set will cause him to favor the perception of one figure to the relative exclusion of other possibilities. In the hidden objects test, the subject is asked to find as many objects as possible in pictures similar to those found in children's magazines. In the Gottschaldt Figures test, the subject is asked to pick out the simple outlines disguised in complicated figures. In the use of these tests, it is assumed that rigid individuals will have difficulty in seeing objects or outlines and will produce a short list.

Aniseikonic lenses, initially designed to study distortions in stereoscopic perception, also have been used to study rigidity (6). The effect of the lenses is "to make a table appear to tip up like a drawing board or to make a wall lean towards the observer." It was noticed that people varied in the time and degree of perception of the distortion and suggested that more rigid individuals, who manipulated the world to conform to their own preconceptions, would report less distortion and take longer to see it.

Ittleson aptly summarizes this topic by saying:

The assumptive world of any particular individual at any particular time determines his perceptions; that is, provides him with predictions of probable significances. His assumptive world is, therefore, in a very real sense, the only world which he knows (26).

*State of the Subject in Relation to Rigidity.* The effect of stress on problem solving has been widely studied, using such diverse stimuli as electric shock, fear of failure on a test, frustration, sudden cold showers, time pressures, and anxiety resulting from the interpretation of the Rorschach (5, 13-14, 16-17, 20). These studies have led

to the generalization that rigid behavior tends to increase in a generally monotonic fashion as a function of increasing psychological stress. In many stress-inducing situations, subjects are unable to deal with problems they feel they should be able to handle. This may make them somewhat defensive, insecure, and no longer able to feel completely free in exploring the test situation. In general, negative attitudes appear to be expressed toward situations of more tense psychological stress.

The stressful impact of any situation, however, doubtless varies considerably from person to person. A situation that is non-stressful to some may be stressful to others, either because they are relatively ill-equipped to deal with it or because of its similarity to other situations in which insecurity was previously experienced. It has been suggested that increasing the difficulty of the problem or the effort required for its solution tends to effect emotional stress and frustration provided the subject is at all ego-involved about reaching the goal. In Einstellung tests, the set formula offers a relatively secure alternative of disorganization, and hence insecure subjects tend to return to it, even though in so doing they avoid tackling the required solution to the problem.

*Age in Relation to Rigidity.* While most theories of rigidity posit a relationship between age and rigidity, some posit a decrease, others an increase, of rigidity with age. Proponents of the former view say, in effect, that the more differentiated the individual, the more capable he is of conceiving of a given situation in a variety of ways, and therefore the less rigid his behavior will be. Since differentiation is assumed to increase with age, rigidity of behavior should decrease monotonically with an increase in age, barring senility. If, according to this view, maturation consists of both the building up of more and more complicated patterns of reaction mechanisms and the increased integration of these patterns, then the immature child can differentiate only imperfectly and, therefore, takes each "stimulation" as a "whole."

Proponents of the view that rigidity increases with age imply that the older person has had more opportunity to develop fixations and that rigidity is a reaction to threat to the ego. Since the ego defense system is more developed in the older individual, rigidity should increase with age. For example, because he found children more susceptible than adults to distortion produced by aniseikonic lenses, Becker considered children less rigid than adults (6). While each view has certain experimental evidence for support, the picture is unclear and no absolute judgment as to the efficacy of either view can be made at this time.

The complexity of the task has been found to affect subjects differentially according to their age (12, 22, 28). While performance deteriorates comparatively slightly with age on simple tasks, the deterioration is generally marked on complex tasks that involve combining simple skills. Older subjects take more time, are less accurate, and less active in correcting errors. "Older" is a relative term, however, and in these studies pertains to adults generally over 55. The results may not, therefore, be applicable for comparisons between "old" and "young" children. In fact, there is increasing evidence for a curvilinear relationship between age and rigidity, with young adults showing less Einstellung-effect than children or older adults.

It has already been mentioned that studies showing older children to be more rigid than younger children may not represent an inherent age-rigidity relationship. Rather, the greater Einstellung-effect shown by older subjects may be due to a longer exposure to schooling that stresses drill and repetition. In this connection, the findings of Cunningham are somewhat in contrast to those of Luchins (16, 32). It was postulated that the finding of an inverse relationship between age and rigidity by the former investigator may have been the result of changes in the curriculum of the elementary school and/or teaching procedures since the late 1930's and early 1940's, the period Luchins conducted his basic experiments. Perhaps curriculum reformers heeded Luchins' persistent warnings of schools' inducing an "educogenic" rigidity.

*Intelligence in Relation to Rigidity.* Common also to several theories of rigidity is the notion that greater intelligence leads to a higher degree of differentiation and, therefore, less rigidity. However, classroom instruction adapted to the intelligence of the pupils is likely to be different and to complicate the study of the relationship between intelligence and rigidity. While both high and low-I.Q. groups studied showed large Einstellung-effects, superficially similar, Luchins theorized that it was brought about by different processes (33). For example, comments suggested that brighter children quickly generalized the Einstellung method as the rule of solution, while low-I.Q. children, more insecure in their arithmetic ability, blindly stuck to the set method once they discovered it. The average correlation between intelligence and rigidity, as measured by the Water Jar Test, has been computed as about  $-.17$  (29).

*Sex Differences in Rigid Behavior.* Some studies have shown males less susceptible to set and better able to overcome an established set than females, although the differences were usually slight and not significant. Other studies have shown no differences. Luchins postu-

lated that the greater Einstellung-effects shown by females was associated with their possessing, to a greater degree than males, such traits as dependency and submissiveness (34).

*The Effect of Speed Conditions on Rigidity.* It is generally agreed that abstract thinking can occur only when there is time to remove oneself sufficiently from the immediate problem situation. The person in a hurry will be forced to perceive the problem confronting him narrowly and frequently to resort to behavioral supports of a concrete nature in order to solve the problem. Conversely, the person in no hurry will have time to perceive the problem more broadly. Generally speaking, experimental results support this theoretical analysis.

One might suppose, however, that if a decrease in time availability effects an increase in rigidity, then an increase in time availability should effect a decrease in rigidity. The situation does not appear to be this simple since when the time factor is explicitly eliminated, no significant decrease in rigidity has been found (40). The fact that subjects are given more time to solve problems does not mean that they utilize the time for that purpose, but, perhaps, use the "extra" time for such post-solution activities as checking computations and writing solutions more neatly. It may be that children quite early learn that all situations calling for responses are speed measures. The extra time sometimes provided for free exploration then becomes phenomenologically superfluous.

*The Effect of Concretizing the Task Upon Rigidity.* That there were subjects who regarded the numbers on the Water Jar Test as essentially abstract symbols was obvious when 62% of a college class failed the following problem: Given a 4-quart jar, a 67-quart jar, and a 17-quart jar, get 4 quarts (33). Several investigators have pointed out that since the traditional paper-and-pencil version of the Water Jar Test merely involves the drawing of arrows from one figure to the others, the continued use of the longer, or "rigid," method of solution when the shorter one is available may actually be more advantageous than to change one's mode of attack (33, 44). Subjects in several studies, therefore, have been provided with actual water and water containers. Inconclusive results have been reported, probably as a result of the use or non-use of scratch paper. When scratch paper has been provided, many subjects apparently figure out the solution on paper and then manipulate the jars in accordance with their calculations, thus defeating the purpose of introducing the actual containers. In one study, the few children who carefully examined each set of containers, treated each problem as possessing individual requirements, and used written calculations only as a check on their manipulations, showed little or no Einstellung-effect (33). It thus appears that a tendency toward mechanization can occur both on

the concrete and abstract levels, the major factor appearing to be the attitude with which subjects view the tasks.

Marks studied problem-solving behavior in two situations—"real life" (direct interaction with the problem) and "textbook" (a story about a fictional character confronted with the same problem) (37). In the latter case, the student was to "advise" the character as to how to solve the problem. Underlying rationale was that in "real life" situations, the person finds himself within the problem context, making it difficult to see himself objectively. Therefore, the person is more likely to show defensive reaction in "real life" than when he is less involved personally. Mark's results were interpreted as supporting such a view. It now seems generally accepted that intense personal involvement can lead to rigid behavior, especially if the situation is at all threatening. An ego threatened by unmanageable situations wards off the threat by limiting the scope of stimuli it will accept and by distorting those stimuli it cannot ignore. However, the resulting restriction in the range of behavior is not entirely maladaptive as it is an effort toward ordering experience. Forcing children to make predictions about the outcome of experiments may, for some, lead to rigid behavior and perception.

#### DISCUSSION

There have been many proposals as to the best way to prepare people for problem-solving tasks. It has been found that flexibility is a type of learned behavior in which the subject anticipates change and seeks to solve problems in alternative ways (1, 41). Of course, subjects trained on repeated presentations of the same problem are generally more proficient in solving new problems of the same class than subjects trained on a variety of problems (2). The latter, however, have a smoother transition to new problems than the former. It has been well substantiated that training on a number of different problems leads to the growth of learning sets, or ideas of how problems should be tackled (25). Maier has even shown that telling students to break their set results in better problem solving (36).

Not all attempts to train for flexibility have been successful, however. Luchins, for example, repeatedly failed to induce flexible thinking in children and concluded that they

... had developed [rigid] attitudes and habits . . . as a result of their schooling. They were accustomed to being taught a method and then practicing it; to have to discover procedures was not only quite foreign to them in arithmetic but also in most school subjects (32).

The value of "drill" activities in problem solving is unclear. Training in a particular solution leads to maximum efficiency but only so long as problems are confined to the class studied. Where a variety

of problems have to be faced, wider training with emphasis on the need for change is advisable. Probably any one type of problem should not be used frequently enough to establish a set. While some evidence shows an average of six to seven similar problems to be sufficient for the establishment of an *Einstellung*-type set, weaker sets are probably induced with even fewer problems. Sets may be induced in more intelligent subjects by fewer problems than are necessary for less intelligent subjects.

One of the unfortunate aspects of "drill" is its tendency to elicit responses that subjects feel are "wanted" by the authority. Several studies showed that students in "progressive" or "modern" schools tended to be more flexible in their problem solving than students in more traditional schools. While other, perhaps antithetical, considerations may be pertinent, permissive teaching atmospheres appear to create better conditions for divergent thinking than do authoritarian atmospheres.

Socratic questioning may be particularly valuable for flexibility in thinking because it gives the student an opportunity to become explicitly aware of his assumptions and helps him to see where his understandings are vague or inadequate. There is evidence that the teacher may effectively combat rigidity by repeatedly pointing out the disadvantages of rigid behavior. Having students search for multiple rather than unitary causes is particularly valuable. Perhaps this is the value of debating, i.e., forcing the participants to take another position or to view a problem from a different angle. In this context it is probably well to avoid lessons purporting to offer "proof" of the cause of a particular event. A general weakness of such demonstrations is that the audience either is convinced of the "correct" outcome at the start or they accept the instructor's description of the outcome without involving their own judgment.

Training on a number of different problems leads to the growth of learning sets. Flexibility may be said to consist of expecting changes and looking for alternative pathways all the time. Inability to use an object or process for a "strange" purpose may be due to its previous and extensive use for a single purpose, a phenomenon known as "functional fixation." This can perhaps best be avoided by analyzing the characteristics of the objects or processes necessary for problem solution and then analyzing the characteristics of the objects and processes available. "Tools" should be associated with their principles and characteristics rather than with a specific use.

Ambiguity may be achieved in a number of ways but always implies that the stimulus itself possesses intrinsic characteristics that make it possible for its being perceived in more than one way. When the stimulus elements are ambiguous, the assumptions and precon-

ceptions of the subject become highly important in determining the response. In such a situation it becomes more economical and safe to perceive and act in a mechanical fashion and rigid behavior is frequently a result. Speeded exposure of material may also be considered an ambiguous situation, even when highly structured material is presented, in the sense that it does not permit the subject to come to full grips with the material offered.

In summary, it seems apparent that the behavior labeled as rigid is potentially determined by neither situational nor personality factors alone but by the interaction of both. Rigidity then does not exist within an individual to be projected into concrete situations nor can the structure of the situation alone elicit rigid behavior. But, since the personality of the individual is generally not modified to any extent by the single task but exists as a persistent variable in determining his behavior, certain consistencies in behavior should be manifested by a given individual even when working at dissimilar tasks. Individuals may, however, show rigid behavior in demanding situations but exhibit none when confronted with tasks which fall in areas of high competence.

As pointed out by Luchins and Lewin, one must be careful not to think in Aristotelian terms, i.e., to use a dichotomous classification in place of a continuous gradation and to classify according to end products rather than according to the nature of the psychological processes involved (31, 34). In other words, not to imply that subjects respond as they do on a rigidity scale because they belong to a class known as rigid persons.

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#### VITAL WATER SYSTEMS TO 'BLANKET' U. S. BY 1972

The entire United States will be blanketed by 1972 with water quality management systems to control cleanliness and supply of the nation's vital resources, a water expert told a special symposium on land and water resources.

Proper management of the quantity and quality of water is urgently needed throughout the growing nation, said Dr. Keith S. Krause, division of water supply and pollution control, U. S. Department of Health, Education and Welfare.

Water systems are now being planned and coordinated by HEW and other federal and state agencies for water resources and pollution.

"A stream functions very much like a freight train or truck," Dr. Krause said. "Its waters can carry a variety of things or a single commodity. If the load is too heavy, something breaks down. If only one water user has a monopoly on it, its general usefulness is greatly curtailed. The efficient stream system is one where the waters accommodate a large number of uses, can be unloaded rapidly and made ready for the next user."

In 1900, the population of the United States was 75 million, 39% of whom lived in urban areas and used about 35 gallons of water per person per day. Today about 70% of our 192 million people are urban dwellers, each of whom use about 150 gallons a day.

By the year 2000 the U. S. population may reach nearly twice what it was even at the last census. At this point the water systems could break down, unless necessary measures are taken, said Dr. Krause.

Plans are now underway to set up water quality management systems in various areas such as the Columbia River Basin, the Great Lakes, the Ohio River Basin, the Susquehanna-Chesapeake Bay area, the Delaware Basin, the Hudson River Basin, the Southeast River Basins, the Missouri River Basin and the Arkansas-Red River Basins.

## Problem Department

Conducted by Margaret F. Willerding  
San Diego State College, San Diego, Calif.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution or proposed problem sent the Editor should have the authors name introducing the problem or solution as on the following pages.

The Editor of the Department desires to serve her readers by making it interesting and helpful to them. Address suggestions and problems to Margaret F. Willerding, San Diego State College, San Diego, Calif. 92115

### SOLUTIONS AND PROBLEMS

Note: Persons sending in solutions and submitting problems for solution should observe the following instructions.

1. Solutions should be in typed form, double spaced, or printed.
2. Problems and solutions should be submitted in the same form as they appear in the journal.
3. Each problem or solution should be submitted on a separate sheet.
4. Drawings in India ink should be on a separate page from the solution.
5. Give the solution of the problem which you propose if you have one and also the source and any known reference to it.
6. In general when several solutions are correct, the one submitted in the best form will be used.

3026. Proposed by Brother Felix John, O'Connell H. S., Arlington, Va.

Show that

- 1) any odd square  $N$  can be represented as the sum of two consecutive positive integers
- 2) any even square  $N$  can be represented as the sum of two positive consecutive odd integers.

*Solution by the proposer*

1) Let

$$N = (2n - 1)^2 = 4n^2 - 4n + 1 = (2n^2 - 2n) + (2n^2 - 2n + 1),$$

which are two consecutive positive integers for  $n > 1$ .

2) Let

$$N = (2n)^2 = 4n^2 = (2n^2 - 1) + (2n^2 + 1),$$

which are two positive consecutive odd integers for  $n \geq 1$ .

Solutions were also submitted by Merrill Barnebey, La Crosse, Wis.; Richard H. Bates, Milford, N. Y.; R. P. Beem, San Francisco, Calif.; W. E. Bunker, Pittsburgh, Pa.; Thomas Vanden Eynden, Athens, Ohio; Herta T. Freitag, Hollins, Va.; Herbert R. Leifer, Pittsburgh, Pa.; H. C. Torreyson, Defiance, Ohio; C. W. Trigg, San Diego, Calif.; and Dale Woods, Kirksville, Mo.

3027. Proposed by Robert A. Carmen, San Bernardino, Calif.

Find the smallest integer such that if the left-most digit,  $n$ , is transposed to the extreme right the new number is  $1/n$  of the original number.